$$6(\sqrt{\delta})' = \sqrt{z} - 2\lambda(\sqrt{z})',$$
$$z' = \begin{cases} 2(\delta - 1), & \lambda > 0 \\ 2(\delta - 1/n), & \lambda < 0 \end{cases}$$

with the boundary conditions (2.5), (2.6). From this it can be deduced that the function z has a discontinuity at $\lambda = 0$. At the point $\lambda = 0$ the density derivative δ' is continuous. The solution can be constructed numerically. The density profile for n = 10 is given in Fig. 1.

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GAS OSCILLATIONS IN A PIPE WITH A NONLINEAR ACTIVE LOAD

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UDC 532.542

Characteristics of the working cycle in the cylinder of a piston compressor lead to the appearance of pressure and velocity oscillations of the medium being transported in the associated manifold system, i.e., a pulsating gas flow. Oscillatory gasdynamic processes in pipes lead to an appreciable decrease in efficiency and reliability in the use of a compressor. One of the most efficient methods of decreasing the effect of a pulsating gas flow is the matching of the initial section of the manifold. The essence of the method consists in fitting a lumped resistance (load) after that part of the manifold which is closest to the cylinder. The magnitude of this load is arranged to be equal to the wave resistance of the pipe [1]. Since the gas oscillations are low-frequency, the load resistance turns out to be active and nonlinear as in a steady-state flow [2]. Hence the possibility of matching with a nonlinear resistance needs to be investigated further.

The one-dimensional nonsteady-state motion of a gas in a round pipe of constant cross section with a velocity of motion much less than the velocity of sound is described by the system of equations [3]

$$\dot{\rho} + w' = 0,\ \dot{w} + (w^2/\rho + \rho^\gamma/\gamma)' + (\lambda/\rho)w|w| = 0\ (0 \leqslant x \leqslant 1). \tag{1}$$

The system (1) has been written in dimensionless form (the length of the pipe, the velocity of sound, and the equilibrium gas density are equal to unity), w is the flow, ρ is the relative density, γ is the polytropic index, and λ is the dimensionless hydraulic resistance coefficient. A point denotes differentiation with respect to time t and a dash, with respect to the x coordinate.

The boundary conditions have the form

$$w = f(\omega t)\ (x = 0);\ (\rho^\gamma - 1)/\gamma = (\eta/\rho)w|w|\ (x = 1), \tag{2}$$

where f is a given periodic function with period 2π, and η is the hydraulic resistance coefficient of the lumped insert in the Darcy–Weissbach formula.

In what follows we shall assume that $\eta \gg 1$, $\lambda \lessdot 1$, while the basic nonlinear effect is associated with the active resistance for x = 1.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 81-85, July-August, 1976. Original article submitted July 15, 1975.

Substituting $w_* = \eta w$, $\xi_* = \eta(\rho - 1)$, $f_* = \eta f$ in Eqs. (1), (2), and neglecting inverse powers of η, we obtain, on omitting the asterisk,

$$\dot{\xi} + w' = 0,\ \dot{w} + \xi' = 0\ (0 \leqslant x \leqslant 1); \tag{3}$$

$$(w)_0 = f(\theta),\ (\xi)_1 = (w|w|)_1\ (\theta = \omega t). \tag{4}$$

The general solution of the linear system (3) is expressed in terms of the function G of argument $\omega(t - x)$ and the function H of argument $\omega(t + x - 2)$ in the form

$$\xi = G + H,\ w = G - H. \tag{5}$$

Setting $F(\theta) = G(\theta) - H(\theta)$, we have, from Eq. (5) and the second condition of Eq. (4),

$$2G = F(|F| + 1),\ 2H = F(|F| - 1). \tag{6}$$

We obtain the following difference equation for determining $F(\theta)$ from Eqs. (5), (6), and the first of conditions (4):

$$2f = F|F| + F - F_-|F_-| + F_-. \tag{7}$$

Here and in what follows we use the notation $\varphi_-(\theta) = \varphi(\theta - 2\omega)$. We also restrict ourselves to finding the $F(\theta)$ with period 2π. We have from Eq. (7)

$$F_- = F = f\ (\omega = \pi n,\ n = 1, 2, 3, \ldots). \tag{8}$$

In the case $\omega = (1/2)\pi n$ ($n = 1, 3, 5, \ldots$) we replace θ by $\theta - 2\omega$ in Eq. (7) for finding F. We then obtain

$$2f_- = F|F|_- + F_- - F|F| + F.$$

It follows from this and from Eq. (7) that

$$f + f_- = F + F_-,\ f - f_- = F|F| - F_-|F_-|. \tag{9}$$

If $f_- = -f$, then

$$f|f|^{-1/2} = -F_- = F. \tag{10}$$

We make the following substitutions in Eq. (9) in the general case:

$$f - f_- = 2\beta|\beta|,\ f + f_- = 2\alpha\beta,\ F = \beta(\alpha + \varkappa),\ F_- = \beta(\alpha - \varkappa). \tag{11}$$

We then obtain the following equation for $\varkappa$:

$$(\alpha + \varkappa)|\alpha + \varkappa| - (\alpha - \varkappa)|\alpha - \varkappa| = 2,$$

which has the solution

$$\varkappa = \begin{cases} \sqrt{1 - \alpha^2}, & 2\alpha^2 \leqslant 1 \\ 1/(2|\alpha|), & 2\alpha^2 \geqslant 1. \end{cases} \tag{12}$$

For $f \to f_-$ Eq. (8) is obtained from Eqs. (11), (12) corresponding to the fact that the excitation frequency is doubled.

By way of an example we consider the case of harmonic excitation

$$f = \varepsilon \cos\theta. \tag{13}$$

We have from Eqs. (5), (8)

$$2w(x, t) = \varepsilon \cos \theta_- + \varepsilon^2 \cos \theta_- |\cos \theta_-| - \varepsilon^2 \cos\theta_+ |\cos \theta_+| +$$
$$+ \varepsilon^2 \cos \theta_+ (\theta \pm = \theta \pm \omega x, \ \omega = \pi n, \ n = 1, 2, \ldots). \tag{14}$$

For $\varepsilon \ll 1$ the solution (14) is close to the harmonic function which is the solution of the linear problem, Eqs. (3), (4), with the condition $(\xi)_1 = 0$. This was to be expected in view of the fact that the frequencies under consideration differ from the eigenfrequencies of the linear problem, and so for $\varepsilon \ll 1$ nonlinear effects are negligible.

For $\varepsilon \gg 1$ the form of the oscillations can cease to be harmonic. To estimate the amplitudes of the higher-order harmonics we represent Eq. (14) in terms of the coefficients of the series [4]

$$\cos \theta |\cos \theta| = \sum_{k=0}^{\infty} a_k \cos (2k+1) \theta, \quad a_k = \frac{8(-1)^k}{\pi (1-4k^2)(2k+3)} \tag{15}$$

in the form

$$2w = (\varepsilon + \varepsilon^2 a_0) \cos \theta_- + (\varepsilon - \varepsilon^2 a_0) \cos \theta_+ + \varepsilon^2 \sum_{k=1}^{\infty} a_k [\cos (2k+1)\theta_- - \cos (2k+1)\theta_+]. \tag{16}$$

It follows from Eq. (15) that the amplitudes of the higher harmonics are relatively small, and so a matching regime is possible.

By analogy with linear waveguides, matching can be defined as the regime in which the velocity equation (16) does not contain the first harmonic of the reverse wave (the coefficient of cos θ_+ is zero). In this case

$$w = \varepsilon \left[\cos \omega (t - x) + \sum_{k=1}^{\infty} \frac{a_k}{a_0} \sin (2k+1) \omega t \sin (2k+1) \omega x \right],$$
$$\varepsilon = \frac{1}{a_0} = \frac{3\pi}{8} \approx 1.18. \tag{17}$$

The velocity amplitude equation (17) differs from a constant because of the small higher harmonics, and so the difference is not large. For frequencies $\omega = (1/2)\pi n$ ($n = 1, 3, 5, \ldots$) we have, from Eqs. (5), (6), (10), and (13),

$$2w = \varepsilon \cos \theta_- + \sqrt{\varepsilon} b(\theta_-) + \varepsilon \cos\theta_+ - \sqrt{\varepsilon}\, b(\theta_+),$$
$$b(\theta) = \cos \theta |\cos \theta|^{-1/2}. \tag{18}$$

By contrast with Eq. (14), when $\varepsilon \ll 1$ the velocity $w \sim \sqrt{\varepsilon} \gg \varepsilon$ and is determined by the nonlinear effects. This is associated with the fact that the frequencies under consideration are resonance frequencies of the linear problem. We now write Eq. (18) in the form of a series, using the expansion [3]:

$$b(\theta) = \sum_{k=0}^{\infty} b_k \cos (2k+1) \theta, \quad b_k = \frac{2(-1)^k}{\sqrt{\pi}} \frac{\Gamma\left(k + \frac{5}{4}\right)}{(4k+1)\Gamma\left(k + \frac{7}{4}\right)},$$

which gives, in the case of matching,

$$\varepsilon = b_0^2 = \frac{4}{\pi} \Gamma^2 \left(\frac{5}{4}\right) \Gamma^{-2} \left(\frac{7}{4}\right) \approx 1.23,$$
$$w = \varepsilon \left[\cos \omega (t - x) + \sum_{k=1}^{\infty} \frac{b_k}{b_0} \sin (2k+1) \omega t \sin (2k+1) \omega x \right]. \tag{19}$$

It is clear from Eqs. (17), (19) that the values of ε are close in the case of matching.

For arbitrary frequencies the problem (3), (4), (13) can be solved approximately by the method of harmonic balance. This problem permits a solution which changes sign when θ is changed to $\theta + \pi$ [this can be seen immediately in the particular cases of Eqs. (14), (18)]. To find this solution we set

$$w(1, t) = \sum v_n e^{in\psi} = v(\psi) \quad (\psi = \theta - \varphi, \ v_{-n} = \bar{v}_n, \ n = \pm 1, \pm 3, \ldots), \tag{20}$$

where φ is chosen so that

$$v(0) = 0, \ -v(\psi + \pi) = v(\psi) > 0 \ (0 < \psi < \pi). \tag{21}$$

We then find from Eqs. (3), (20) and the first* of conditions (4)

$$\xi = \sum_m \frac{ie^{im\psi}}{\sin m\omega}\left[v_m \cos m\omega x - \frac{1}{2}\varepsilon\delta_{|m|1}e^{im\varphi}\cos m\omega(1-x)\right];$$
$$w = \sum_m \frac{e^{im\psi}}{\sin m\omega}\left[v_m \sin m\omega x - \frac{1}{2}\varepsilon\delta_{|m|1}e^{im\varphi}\sin m\omega(1-x)\right]. \qquad (22)$$

From the second boundary condition (4) we have

$$\xi(1, t) = v|v| = \sum_m e^{im\psi}\left(\frac{2i}{\pi}\sum_{n,p}\frac{v_n v_p}{n+p-m}\right). \qquad (23)$$

We have from Eqs. (20), (21), and comparison of the coefficients of Eqs. (22), (23)

$$v_m \cos m\omega - \frac{\varepsilon}{2}\delta_{m1}e^{i\varphi} = \frac{2}{\pi}\sin m\omega \sum_{n,p}\frac{v_n v_p}{n+p-m},$$
$$\sum v_n = 0 \quad (v_{-m} = \bar{v}_m, \quad m = 1, 3, 5, \ldots). \qquad (24)$$

Only the first harmonic is taken into account in the harmonic approximation. It follows that $v_1 = -ir$ from the equation $v_1 + v_{-1} = 0$, and for real r, φ with $m = 1$ we have from Eq. (24)

$$(16/3\pi)(\sin\omega)r^2 - i(\cos\omega)r = (\varepsilon/2)e^{i\varphi}. \qquad (25)$$

Whence

$$[(16/3\pi)r^2 \sin\omega]^2 + r^2\cos^2\omega = \varepsilon^2/4, \quad \mathrm{ctg}\,\varphi = -(16/3\pi)r\,\mathrm{tg}\,\omega. \qquad (26)$$

When Eq. (25) is taken into account, the first harmonic of the velocity Eq. (22) is equal to

$$(r + (16/3\pi)r^2)\sin(\psi - \omega x + \omega) + (r - (16/3\pi)r^2)\sin(\psi + \omega x - \omega).$$

For $r = 3\pi/16$ we have matching. In this case from Eq. (26) $\varepsilon = 2r = 3\pi/8$ for any ω. The resulting value of ε coincides with the exact value Eq. (17), and differs only slightly from Eq. (19). Thus the harmonic approximation is accurate enough.

Finally, we should note the dependence of amplitude on frequency for different values of ε. If $\varepsilon \ll 1$, then the maximum amplitude is $\sim\sqrt{\varepsilon}$ and is attained at the resonance frequencies $\omega = (1/2)\pi n$ ($n = 1, 3, \ldots$), while the minimum amplitude is $\sim\varepsilon$ at the frequencies $\omega = \pi n$ ($n = 1, 2, \ldots$). For $\varepsilon \sim 1$ the amplitude is only feebly dependent on ω. For $\varepsilon \gg 1$ the maximum amplitude is $\sim\varepsilon^2$ and is attained at frequencies $\omega = \pi n$ ($n = 1, 2, \ldots$), while the minimum is $\sim\varepsilon$ and occurs at the resonance frequencies.

The properties of the amplitude noted above are connected with the change (due to nonlinearity) in the very nature of the boundary condition. For $\varepsilon \to 0$ the end of the pipe is almost open ($\xi_1/w_1 \to 0$, since $w_1 \to 0$), while for $\varepsilon \to \infty$ the pipe is almost closed. For intermediate values $\varepsilon \sim 1$ a matching regime is achieved in which a wave of the fundamental frequency reflected from the end is absent, and the amplitudes of the higher harmonics are small.

This treatment of gas oscillations in a pipe with nonlinear resistance allows us to make the following remarks:

1) the dependence of the oscillation amplitude on frequency changes appreciably as the oxidation amplitude is varied (in this respect the mean-square resistance is strongly nonlinear),

2) distortions of harmonically excited oscillations are relatively small (hence a matching regime is possible). In this respect the mean-square resistance is weakly nonlinear.

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THEORETICAL DESCRIPTION OF THE PHENOMENON OF LOSS OF FLUIDITY IN POLYMER LIQUIDS SUBJECTED TO INTENSIVE DEFORMATION

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UDC 532.5:532.135

Polymer liquids display a number of properties characteristic of solids: slippage along the wall, the appearance of cracks in the material during flow, brittle fracture under tension, etc. The combination of these phenomena encountered in the flow of polymers through capillaries, accompanied by a number of other effects (oscillations and waves on the surface of a jet emerging from a capillary, crystallization of polymers in a capillary, etc.), has been referred to in the literature as "destruction of melt." The bibliography devoted to this question, which is important for many polymer-processing methods, is very extensive (see, for example, [1]). The behavior of polymer liquids has been observed recently for melts of polymers with a narrow molecular-weight distribution (MWD) [2, 3] for conventional types of deformation. In [4] the hardening effect was studied for the case of the extraction of polyoxyethylene from a tank by means of a rotating drum. The length of the liquid jets so obtained was as much as half a meter. In the present study we propose a theoretical description of the above-mentioned effects for two of the situations most often found in practice: simple shear and simple tension.

§1. A theoretical description of the phenomenon of loss of fluidity in polymer liquids and their transition to a highly elastic state will be considered in the simplest three-constant nonlinear model of an elastoplastic medium of Maxwellian type, proposed in [5],

$$\boldsymbol{\sigma} = -p\boldsymbol{\delta} + 2\mathbf{C}W_1 - 2\mathbf{C}^{-1}W_2 \quad (W_j = \partial W/\partial I_j); \tag{1.1}$$

$$\mathbf{C}^{\nabla} - \mathbf{C}\mathbf{e} - \mathbf{e}\mathbf{C} + 2\mathbf{C}\mathbf{e}_p(\mathbf{C}) = 0, \ \mathrm{sp}\,\mathbf{e} = 0, \ \det \mathbf{C} = 1; \tag{1.2}$$

$$\mathbf{e}_p = (2\lambda_*(T))\exp\{-(\beta/\mu_0)W_s\}[(\mathbf{C} - \boldsymbol{\delta} I_1/3)W_{s,1} - (\mathbf{C}^{-1} - \boldsymbol{\delta} I_2/3)W_{s,2}]; \tag{1.3}$$

$$I_1 = \mathrm{sp}\mathbf{C}, \ I_2 = \mathrm{sp}\mathbf{C}^{-1}, \ W = \rho_0 f(T, I_1, I_2), \ 2W_s = W(I_1, I_2) + W(I_2, I_1); \tag{1.4}$$

$$D = \frac{1}{3\lambda_*(T)}\exp\left\{-\frac{\beta}{\mu_0}W_s\right\}\{(I_1I_2 - 9)(W_1W_{s,2} + W_2W_{s,1}) + 2(I_1^2 - 3I_2)W_1W_{s,1} + 2(I_2^2 - 3I_1)W_2W_{s,2}\}, \tag{1.5}$$

where $\mathbf{C}^{\nabla} = (\partial/\partial t + v_\alpha \partial/\partial x_\alpha)\mathbf{C} + \omega\mathbf{C} - \mathbf{C}\omega$.

Here we give the rheological equations of the model (1.1)-(1.3) for an incompressible liquid in a Cartesian coordinate system; $\boldsymbol{\sigma}$ is the stress tensor; p is the isotropic pressure; **e** is the tensor of deformation rates; ω is the vortex tensor; the symmetric positive-definite tensor **C** represents the elastic deformation (Finger measure) accumulated during the motion of the elastic liquid; e_p is the tensor of the irreversible rate of deformation; $\boldsymbol{\delta}$ is the unit tensor; I_1, I_2 are independent variants of the tensor **C**; f is the specific free energy; W is the elastic potential; D is the dissipation function; $\mathbf{C}^{\nabla}$ is the Jaumann derivative of the tensor **C** with respect to time; and T is the temperature.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 86-93, July-August, 1976. Original article submitted July 8, 1975.